

How to observe the localization law $\sigma(\omega) \propto -i\omega$ for conductivity?

I. M. Suslov

Kapitza Institute for Physical Problems,
Moscow, Russia

The Berezinskii localization law $\sigma(\omega) \propto -i\omega$ for frequency-dependent conductivity was never questioned from the theoretical side, but never observed experimentally. In fact, this result is valid for closed systems, while most of actual systems are open. We discuss several possibilities for observation of this law and experimental difficulties arising at this way.

It is well-known [1, 2] that the electron states in disordered systems can be extended or localized. In the latter case, when a system is an Anderson dielectric, its frequency-dependent conductivity is believed to obey the Mott law, $\text{Re } \sigma(\omega) \propto \omega^2 \ln^\alpha \omega$. In fact, in the low-frequency limit conductivity is dominated by its imaginary part, and dependence $\sigma(\omega) \propto -i\omega$ is expected.

The localization law $\sigma(\omega) \propto -i\omega$ was predicted by Berezinskii in 1973 [3] for one-dimensional disordered systems. According to self-consistent theory by Vollhardt and Wölfle [4], the same result is valid in the localization phase for systems of arbitrary dimension d . In the recent paper [5] of the present author the same behavior of conductivity was established for systems of finite size L at the arbitrary extent of disorder. The latter is a consequence of the fact that a finite system is topologically zero-dimensional, and its effective dimensionality is less than lower critical one ($d_{c1} = 2$ [6]).

The Berezinskii law was never questioned in the theoretical community; however, it was never observed experimentally. This paradoxical situation was clarified in Ref.5: Berezinskii's result is valid in closed systems, while most of actual systems are open. In open systems, replacement $-i\omega \rightarrow -i\omega + \gamma$ occurs (where γ is inelastic damping) and dependence $\sigma(\omega) \propto -i\omega$ transforms into the usual metallic behavior.

A possibility of realization of closed systems became clear after observation of the persistent current in disordered systems (in the Aharonov-Bohm geometry) [7, 8, 9], in accordance with its prediction in [10]. In fact, the persistent current is a consequence of the Berezinskii law, establishing the dissi-

pativeless character of conductance. Its observation is possible, when a size L of the disordered ring is small in comparison with the inelastic length L_{in} , depending on temperature T . The typical scales in the indicated experiments were $L \sim 1\mu m$, $T \sim 100mK$. If one accepts that $L_{in} \propto T^{-2}$ (as for e-e interaction), then a system is closed for $L \lesssim 10nm$ in the helium region ($T \sim 1K$).

Let discuss several experimental situations, where observation of the Berezinskii law is possible.

1. The first variant is the island film of a disordered metal lying on the dielectric substrate (Fig.1). We suppose for clearance that all islands are of the same size L , which increases monotonically in the course of the film deposition¹. Then for $L \lesssim L_{in}$ the Berezinskii law is valid (Fig.1,a), while in the opposite case $L \gtrsim L_{in}$ the usual metallic conductivity takes place (Fig.1,b). A transition from one regime to another can be provided by the change of L or the temperature.

At first glance, the described experiment is simple. However, there is a bottleneck in it. It is clear from relation $\epsilon \sim i\sigma/\omega$, that behavior $\sigma \propto -i\omega$ corresponds to the frequency-independent permittivity ϵ , so a disordered system is an ordinary dielectric. The properties of the film in the Berezinskii law regime are the same as those of the dielectric substrate, hence the former gives a negligible contribution to conductivity in comparison with the latter. The width of the film is by 6–7 orders less than the width of the substrate, but the corresponding smallness can be partially compensated by a large value of the film permittivity ϵ_1 in comparison with

¹ In fact, there is a distribution of islands in size, which shifts to the large L region in the course of deposition.

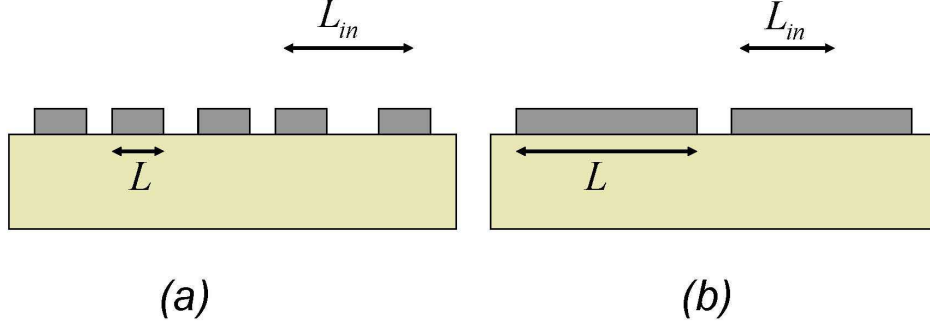


Figure 1: In the case of the island metallic film, the Berezinskii law is observable when the island size L is small in comparison with the inelastic length L_{in} (a), while in the opposite case the metallic behavior is valid (b).

its substrate value ϵ_0 . By the order of magnitude, $\epsilon_1 \sim \xi^2/a_0^2$ (where ξ is the localization length for wave functions, and a_0 is the atomic space), and saturates by a value L^2/a_0^2 for large ξ . If the metallic film is weakly disordered², then for $L \sim 10nm$ its permittivity ϵ_1 can exceed ϵ_0 by 3–4 orders.

The experimental procedure looks as follows. The experiment is carried out *in situ* and begins with a measurement of the frequency and temperature dependencies of the substrate conductivity, with saving results in a file. Then a small amount of metallic atoms is deposited, and again conductivity is measured and saved; again deposition is made on. Proceeding by small steps, one should reach a regime, when the film contribution is clearly seen in the substrate background. Then the actual measurements can be made.

2. The second example is the nanocomposite system [12, 13], which is a dielectric sample with the metallic granules embedded in it (Fig.2). The volume fraction p of a metal can be rather large and its effect should be easily observable, being of the order of unity. However, a "hidden rock" is present here. Let exploit the formula from the Landau and Lifshitz book [15]

$$\frac{\bar{\epsilon} - \epsilon_0}{\epsilon_0} = p \frac{3(\epsilon_1 - \epsilon_0)}{2\epsilon_0 + \epsilon_1}, \quad (1)$$

which is valid for a small concentration of spherical granules: it gives the average permittivity $\bar{\epsilon}$ (for the

² The films are weakly disordered in the case of "simple" metals (such as Mg, Al, Sn), which are well-described by the pseudopotential theory [11]; a small pseudopotential provides weak scattering even in the amorphous state. Contrary, the films of the transition metals are usually strongly disordered.

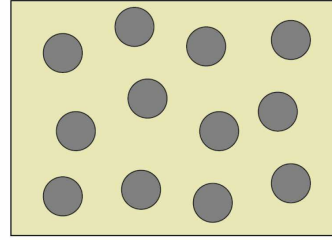


Figure 2: A nanocomposite system with spherical metallic granules embedded in a dielectric.

system of Fig.2) in terms of its values for a dielectric (ϵ_0) and a metal (ϵ_1). Since $\epsilon_1 \gg \epsilon_0$, then

$$\frac{\bar{\epsilon} - \epsilon_0}{\epsilon_0} \approx 3p - 9p \frac{\epsilon_0}{\epsilon_1}, \quad (2)$$

and the main contribution $3p$ is an uninteresting constant, while the useful effect, depending on ϵ_1 , is determining by two small parameters p and ϵ_0/ϵ_1 . As a result, the problem of a reference arises, i. e. a necessity to have the identical sample without metallic granules. Fortunately, such a problem is absent for a specific technology [12, 13, 14], when nanocomposites are produced on the base of a porous glass, whose pores are filled by metallic granules (of suitable size about $7nm$); so the same sample can be measured in absence and in presence of granules. It is useful to note that for the system of Fig.2 (in contrast to that of Fig.1) the strongly disordered metal is desirable, in order to increase the ratio ϵ_0/ϵ_1 .

3. Derivation of Eq.1 is based on a solution of the well-known problem on a dielectric ball in the

external electric field [15]. The analogous problem is solvable for an ellipsoid with arbitrary ratios of its semi-axes a, b, c [15], and generalization of (1) is possible for granules of ellipsoidal form:

$$\frac{\bar{\epsilon} - \epsilon_0}{\epsilon_0} = p \frac{\epsilon_1 - \epsilon_0}{A\epsilon_0 + B\epsilon_1}, \quad (3)$$

where $A = 1 - B$, and

$$B = \frac{abc}{2} \int_0^\infty \frac{dx}{(x+a^2)^{3/2}(x+b^2)^{1/2}(x+c^2)^{1/2}}, \quad (4)$$

if the electric field \mathbf{E} is directed along the axis a .

In reality, the metallic granules are not strictly spherical. For modelling of such situation, one can suggest that granules are ellipsoids with fluctuating ratios of semi-axes. Then for $\epsilon_1 \gg \epsilon_0$ one has

$$\frac{\bar{\epsilon} - \epsilon_0}{\epsilon_0} \approx p \langle B^{-1} \rangle - p \langle B^{-2} \rangle \frac{\epsilon_0}{\epsilon_1} \quad (5)$$

($\langle \dots \rangle$ is averaging over fluctuations), so the structure of Eq.2 is preserved but the coefficients are changed.

Parameter B decreases when a becomes greater than b and c . In the limit of a strongly oblong ellipsoid ($a \gg b \sim c$) one has $B \rightarrow 0$ and Eq.3 takes a form

$$\frac{\bar{\epsilon} - \epsilon_0}{\epsilon_0} = p \frac{\epsilon_1 - \epsilon_0}{\epsilon_0}, \quad (6)$$

i. e. optimal conditions for observation correspond to the needle-shaped granules (Fig.3). In this case one can provide a sufficient smallness of p (which is necessary for validity of Eq.3 and a transparent interpretation of the experiment) and its compensation by a large parameter ϵ_1/ϵ_0 . As a result, the effect is of the order of unity, or even more. Such systems can be fabricated on the base of chrysotile asbestos, which is a stack of the parallel nanotubes [16] with a typical pore diameter $5nm$; since the length of the granules should be essentially greater³, one is induced to work in the millikelvin range of temperatures.

4. In relation with the latter, we can indicate one exotic possibility. If a vessel with superfluid helium is rotated, then a set of the parallel vortices arises. If metallic atoms are injected in helium, they are localized at the vortex cores and form nanowires [18]. Regulating the length of the latter, one can create the desired system (Fig.4). A concentration of the

³ This length can reach $1mm$ [17].

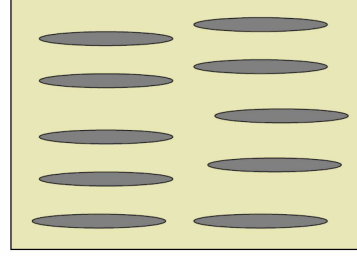


Figure 3: A nanocomposite system with needle-shaped granules.

metallic phase is strongly restricted in this case⁴, but at sufficiently low temperatures one can deal with large L scales and, as a consequence, with enormous values of permeability ϵ_1 .

Analogously, parameter B tends to zero in the case of pancake-shaped granules ($a \sim b \gg c$), if their plane is oriented along the electric field; this case is also described by formula (6). In particular, it is valid in the situation of Fig.1, where the volume concentration p is inevitably small.

In conclusion, the Berezinskii law was not observed previously, since it refers to closed systems, while most of actual systems are open. We have suggested several possibilities for its observation, and shown that experimental difficulties are present in all situations. The latter is rather natural, since in the opposite case this law would be discovered experimentally long ago.

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References

- [1] P. W. Anderson, Phys. Rev. **109**, 1492 (1958).
- [2] N. Mott, E. A. Davis, Electronic Processes in Non-Crystalline Materials, 2nd edition (Oxford: Oxford Univ., 1979)
- [3] V. L. Berezinskii, Sov. Phys. JETP **38**, 620 (1974) [Zh. Eksp. Teor. Fiz. **65**, 1251 (1973)].

⁴ For realistic conditions, one can have about 10^4 vortices per $1cm^2$, while a diameter of nanowires varies from a_0 till several nanometers.

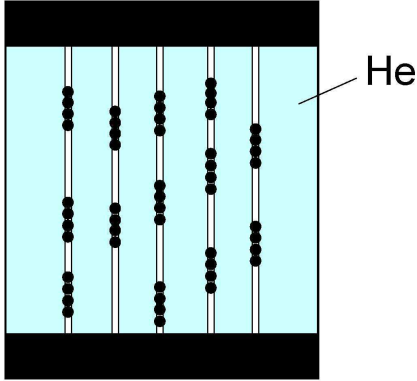


Figure 4: An exotic realization of the system represented in Fig.3. If a vessel with superfluid helium is rotated, then a set of the parallel vortices arises, and the injected metallic atoms are localized on the vortex cores.

- [4] D. Vollhardt, P. Wölfe, Phys. Rev. B **22**, 4666 (1980); Phys. Rev. Lett. **48**, 699 (1982).
- [5] I. M. Suslov, JETP **115**, 897 (2012) [Zh. Eksp. Teor. Fiz. **142**, 1020 (2012)].
- [6] E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).
- [7] L. P. Levy, G. Dolan, J. Dunsmuir, H. Bouchiat, Phys. Rev. Lett. **64**, 2074 (1990).
- [8] H. Bluhm, N. Koshnick, J. Bert, et al, Phys. Rev. Lett. **102**, 136802 (2009).
- [9] A. C. Bleszynski-Jayich, W. E. Shanks, B. Peaudecerf, et al, Science **326**, 272 (2009).
- [10] M. Buttiker, Y. Imry, R. Landauer, Phys. Lett. A **96**, 365 (1983).
- [11] W. A. Harrison, Pseudopotentials in the Theory of Metals, Benjamin, New York, 1966.
- [12] I. V. Golosovsky, R. G. Delaplane, A. A. Naberezhnov, Yu. A. Kumzerov, Phys. Rev. B **69**, 132301 (2004).
- [13] G. Kh. Panova, A. A. Naberezhnov, A. V. Fokin, Physics of the Solid State, **50**, 1370 (2008). [Fizika Tverdogo Tela **50**, 1317 (2008)].
- [14] S. B. Vakhrushev, I. V. Golosovskii, E. Yu. Koroleva, et al, Physics of the Solid State, **50**, 1548 (2008). [Fizika Tverdogo Tela **50**, 1489 (2008)].
- [15] L. D. Landau, E. M. Lifshitz, Electrodynamics of Continuous Media, Pergamon, Oxford, 1960.
- [16] E. Mamontov, S. B. Vakhrushev, Yu. A. Kumzerov, et al, Solid State Communications **149**, 589 (2009).
- [17] S. V. Zaitsev-Zotov, Yu. A. Kumzerov, Yu. A. Firsov, et al, JETP Letters **77**, 1209 (2003).
- [18] E. B. Gordon, A. V. Karabulin, V. I. Matyushenko, etc., JETP **112**, 1061 (2011) [Zh. Eksp. Teor. Fiz. **139**, 1209 (2011)].